VALUATION OF BANK CORPORATE CLIENT RELATIONSHIPS
AS A SWITCHING OPTION

This paper investigates the potential for measurement and identification of value produced by the difference between the volatility of a bank’s existing clients and average client volatility within the industry, using a real options framework. If a bank has loyal clientele, it has effectively created an option to maintain the relationships or exchange them for alternative relationships with clients of other banks. In an industry where the average relationship is profitable, the right to abandon a well-understood relationship that is unprofitable for one that is expected to be, on average, profitable may be quite valuable.

The idea that volatility can create value is antithetical to deterministic valuation methods of relationship value, such as those proposed by Blattberg and Deighton (1996) and Doucet and Barefield (1998). If relationship volatility is seen as a factor that can only reduce the present value of cash flows that are contributed by a relationship, volatility must be a bad thing. Options theory proposes that volatility affects both tails of a probability distribution, and when an option confers a right to a benefit that results from an outcome in the left tail of the distribution (the upside) and/or avoid the effects of an outcome in the right tail (the downside) more volatility increases the probability of a good outcome. If a bank’s relationship with its corporate clients can be described as an option-like contract, it is possible that volatility can produce incremental value.

The assets of interest to this paper are corporate client relationships with their main relationship bank. A bank that is successful in retaining clients may be said to control a valuable asset: Relationships in banking, as defined by Boot and Thakor (1994) and Boot (2000) are created when the bank makes a special commitment when originating the relationship in exchange for future business. The value of the relationship requires a contingent commitment on the part of the client, a commitment which may be valued within a contingent commitment model.

In addition to the present value of future relationship returns, a client with a lower than average propensity to switch (as a result of the contingent commitment) provides the bank with an implicit opportunity: The bank has the option to revise its portfolio of clients, either keeping the existing, loyal client, or abandoning the client in order to free-up capital so that it can take on a new client. The option has value whether exercised or not: Not only is a client worthwhile as a result of the incomplete contract that has created the contingent commitment, but a bank with loyal customers has a valuable right to abandon them. The last statement may sound somewhat cynical, but the option needs not be exercised to have value, and the possibility that such an option exists is sufficient to allow us to estimate its value. Clearly, systematic exercise of an abandonment option would lead to substantial costs to reputation, and literal application of the option pricing model would have consequences that could outweigh the value identified by the real options framework.
The Research Problem

Identification of the incremental value that volatility contributes to bank relationships is the focus of this paper. For this value to be possible, it is necessary that relationship volatility is non-random and that management actions may influence client relationship intentions.

A relationship can be seen as a form of contingent commitment, with at least part of the value derived from a stochastically defined propensity to switch, or in other words, inherent volatility. Volatility confers opportunity, but only when the firm can expect a net gain from switching. A high propensity to switch in the industry not only increases the possibility of acquiring new customers, but more significantly, it makes the value of loyal customers even greater. Thus we are interested in not only the volatility of relationships, but also the covariance of individual bank’s clients’ propensity to switch with the industry's clients’ propensity to switch.

If a financial institution is not constrained in its ability to add new loans, a new relationship will have a profitability profile like the one represented in Figure 1: Once a share of the general cost of advertising and promotion required to acquire new relationships (a) has been covered, the value of the option on the relationship will rise in proportion to the value of the relationship, and the option will always have positive value, because the institution will only exercise the option while the present value of the client relationship (s) is greater than the cost of exercising the option (x).

**Figure 1**  
_A potential new relationship as a call option_

If ‘a’ is the overhead cost of acquiring the new customer and the bank will pay up to the value of the call option to have an option to acquire the new customer, the call option and the acquisition investment should be the same. For example: A bank’s investment in promoting its distinctive competence in a specialised product area (a), divided by the number of potential clients might be $100. The present value of a new relationship (s) net of the direct costs of exercising the option (x) is $120. Since the value of the option is positive, the bank should exercise it, earning a profit of $20 in excess of the average acquisition cost. Even if the option value was less than the overhead costs involved in acquiring the customer the option would have value, since the acquisition investment is a sunk cost.
A relationship in the financial services industry context is different from that of a relationship in a typical manufacturing or retail industry context. Because lending is usually a key part of the relationship, and banks are constrained in the amount of new loans that they can make by capital adequacy rules, at the limit\(^1\), a new relationship may only be created at the cost of an existing one, and the bank must exchange loans rather than create new ones\(^2\). In order to value a relationship, it is easy to appreciate that a new relationship may have value itself, but under conditions of capital constraint the total option value also comes from the right to abandon an existing, less attractive relationship.

Again using the options analogy, the present value of an existing relationship may be likened to a long position in a stock or commodity. (The straight line in Figure 2) The value of the relationship increases in proportion to the increase in the present value of returns to the relationship. In Figure 2, the right to maintain or abandon a client may be seen as a put option that is embedded in the relationship, where the institution will retain the client at its discretion while the present value of the relationship is still seen to be positive with the payment of a retention fee \(r\). (The cost of maintaining a less active or marginal account, for example) The value of the option becomes greater as the value of the relationship becomes negative, as the value of shifting capital from a less attractive client to a more attractive client increases. The put option value should be equal to or greater than the retention cost, or else the option would not be worth retaining.

![Figure 2](image)

\(p = \max \{x-s, 0\}\)

Original relationship

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\(^1\) Assuming that a bank is operating at its optimal capital adequacy ratio, the rate at which the bank can expand its stock of loans and still maintain its capital adequacy ratio is known as the internal capital generation rate, (ICGR), which may be expressed: \(\text{ICGR} = \frac{1}{\text{capital ratio}} \times \text{return on assets} \times \text{earnings retention ratio}\) (Fraser, Gup and Kolari (1995))

\(^2\) For simplicity it has been assumed that the bank does not have access to secondary debt markets and is unable to raise capital through loan sales or securitisation. In theory, access to capital through sources like these can only be undertaken at higher levels of credit risk and at the limit will be impractical.
In Figure 3 these options are combined in a switching (or exchange) option, of the type discussed below. For a bank whose ability to make new loans is constrained by capital controls, entering into a new lending relationship may require the abandonment of an existing relationship. The ability to choose between the existing relationship and the new one requires stability in the existing relationship, an opportunity to attract at least an average-quality new customer in market where information about customers and suppliers is reasonably efficient. The switching option may be represented:

**Figure 3  The value of an option to exchange one client for another**

This combination of options becomes valuable when the present value of the existing relationship falls (and the value of the put option rises) or the present value of the alternative relationship increases. (With the value of the call option becoming greater) If the present value of the relationship is certain to remain at x, there is no prospect of exercising either option, and the switching option has no immediate intrinsic value. As uncertainty increases, and the prospects of exercising the switching option increases, the option becomes more valuable.

In this option framework, uncertainty creates value. At a more intuitive level, it is easy to see that attempting to recruit new customers is pointless in a market with absolute loyalty: Other firms’ clients have to be free and willing to move if the option is to have value and the more willing they are to move, the easier it is for a competitor to attract them. The degree of uncertainty is an important part of the Margrabe option pricing model that is introduced below, and as is shown in Figure 4, as the degree of volatility increases the probability of exercising the option increases, so does the option value.

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3 Sharpe (1990), while discussing the role of information asymmetries in relationship banking, identified three main classes of clients: Own bank profitable and unprofitable clients and other bank clients. While it is possible to identify own client profitability, information asymmetries make it impossible for an outside bank to recognise profitability of competitor clients. For the purposes of the switching option and this paper it has been assumed that all relationships are equally profitable. In practice, the bank would choose to abandon less profitable own clients, but would not be capable of recognising superior profitability in the call option on the average industry client. Dropping the assumption of equal profitability (allowing the bank to abandon less profitable clients first) can only increase the value of the option.

4 The relationship of volatility and option value is well understood and is documented later in this paper as well as in texts like Hull (2003).
Volatility in this model is defined as the uncertainty of returns from an existing relationship, plus the uncertainty of returns from relationships foregone, less the set of interrelationships between these two sources of volatility\(^5\). In Figure 4, the volatility is represented as the normal distribution of returns around the mean value \(x\), the most likely level of relationship returns. In the low-volatility case \(\sigma_2^2\), the likelihood of exercising the switching option is much lower than in the higher-volatility case \(\sigma_1^2\) as much more of the latter’s volume is distributed in the tails. The increase in value of a switching option in a more volatile market is therefore greater by roughly the area of the two banana-shaped segments under the distribution \(\sigma_1^2\) and above the distribution \(\sigma_2^2\).

Thus, it is possible that the volatility of a relationship may contribute a significant part of the determination of the value in the relationship. While relationship value may be affected by the cost of entering into the relationship, the benefit of the relationship, (the present value of future returns to the relationship) and the longevity of the relationship ‘option’, the volatility and sources of volatility are the focus of this paper.

\(^5\) More specifically, volatility of the bank’s own client cash flows is treated as the annualised standard deviation of the continuously compounded natural logarithm of the relationship total returns. The average industry client’s volatility is the annualised standard deviation of the continuously compounded natural logarithm of potential relationship total returns. This is discussed in Chapter 7.
An options pricing analogy may provide a basis for valuing the contribution of a less-volatile-than-average client relationship to its relationship bank in a world where alternative clients present an opportunity for the bank to switch clients.

Where existing clients and alternative new clients are assumed to be equally profitable, from the bank’s perspective, relationships can be valued as an option to exchange one customer for another: the option only exists where the bank’s own customer volatility is lower than average industry volatility.

Methodology

Margrabe’s (1978) partial differential equation approximation of a European exchange option provides the pricing mechanism for this model. Margrabe adapted the pricing of European call and put options to specific application in an option to exchange one risky asset for another. He presented a model of the value of an option \( w \) to exchange one asset \( X_1 \) for another \( X_2 \) exercised only at time \( t \), and therefore a ‘European’ option:

\[
\begin{align*}
w(X_1, X_2, t) &= X_1 N(d_1) - X_2 N(d_2) \\
d_1 &= \frac{\ln \left( \frac{X_1}{X_2} \right) + \frac{1}{2} \sigma^2 (\tau - t)}{\sigma \sqrt{\tau - t}} \\
d_2 &= d_1 - \sigma \sqrt{\tau - t}
\end{align*}
\]

{Equation 1}

In the model, the other pricing factors that are relevant, aside from the price of the two assets being traded are:

- Time remaining until the option expires: \( \tau - t \)
- The difference in volatility between the two assets: \( \sigma^2 \) (explained below)

The model is a product of risk-neutral valuation, the key property of the Black-Scholes methodology that presumes individual actors’ risk preferences are not significant, and returns to an investment are log-normally distributed, with the cumulative probability distribution \( N(x) \) of returns with a mean 0 (abnormal returns or losses are not predicted) and a standard deviation of 1. (Hull (2003)) In the model above, \( N(d_1) \) is the expected value of \( X_1 \) in a cumulative normal probability distribution when \( X_1 > X_2 \), (and zero otherwise): \( N(d_2) \) is the expected value while \( X_2 > X_1 \).

Barone-Adesi and Cyr (1999) expanded on the model by adding an additional term, \( \delta \) to represent a maintenance cost associated with one of the assets (in this case the resource development opportunity). If Margrabe’s model is adapted for this continuous-dividend – like term it would be presented:

\[
\begin{align*}
w(x_1, x_2, t) &= e^{-\delta T} x_1 N(d_1) - x_2 N(d_2) \\
d_1 &= \frac{\ln \left( \frac{x_1}{x_2} \right) + \frac{1}{2} \left( \sigma^2 + \delta \right) (\tau - t)}{\sigma \sqrt{\tau - t}} \\
d_2 &= d_1 - \sigma \sqrt{\tau - t}
\end{align*}
\]

{Equation 2}
Since this model prices the exchange option as a European option and most real options allow early exercise, the ‘European’ nature of the Black-Scholes model appears to be a problem. Margrabe argues that the model is capable of estimating a minimum value for the option, and makes the usual argument that early exercise of an American option would almost always be non-productive and as a result the American option would always be equal in price to a European option. This is also explained by Black (1975).

In order to apply the relative difference in volatility to the exchange option presented in Equations 1 and 2 above, it will be necessary to account for the significant multiple sources of volatility.

The volatility of an industry ‘average’ client and a ‘good bank’s’ clients is thus treated as a product of the difference in the individual paths within two the structural models. The ‘good bank is characterised as having lower-than-average volatility of clientele, and therefore lower-than-average cash flow volatility as a result of fewer outright defections, lower client propensity to deal with multiple banks, and lower client propensity to transfer business to non-relationship bank alternative providers.

If overall relationship volatility is therefore a product of the relative difference in switching intention, given that switching intention is a product of antecedent variables directly, as well as via the firms’ propensity to move business to other banks or additional banks, the option volatility required to solve Equation 2 can be modelled:

\[ \sigma^2 = \sigma_A^2 + \sigma_G^2 - 2\rho_{AG}\sigma_A\sigma_G \]  

{Equation 3}

Where:

- \( \sigma_A^2 \) = an average industry level of volatility of cash flows to a bank relationship
- \( \sigma_G^2 \) = the level of volatility of cash flows to a 'good' relationship bank’s relationships
- \( \rho_{AG} \) = correlation coefficient for the industry and banking sample populations = \( \frac{\sigma_{AG}}{\sigma_A \sigma_G} \)

If the volatilities for \( \sigma_A^2 \) and \( \sigma_G^2 \) are provided from individual bank and industry data, and the correlation coefficient of the two terms may be calculated from the two samples, the value of \( \sigma^2 \) may be calculated and used within the switching option model described in Equation 2.

Black and Scholes (1973) demonstrated that an increase in the variance of an asset always increases the price of the option. Stulz (1982), created a case where an increase in the variance of one of two risky assets did not always increase the value of the option, a proposition that is important here. Stulz argued that the sign of the partial derivative of the option price with respect to the variance of either one of the assets could be ambiguous.

An example may help make this clearer. Starting from a proposition that the option has an exercise price of zero, at expiry \( \tau \), Stulz sets the value of the option equal to:

\[ w(X_1, X_2, 0, \tau) = X_2 - E(X_2, X_1, \tau) \]  

{Equation 4}

The change in option value with respect to a change in volatility in asset \( X_i \) would therefore be:
\[
\frac{\partial w(X_1, X_2, 0, \tau)}{\partial \sigma_{x_1}} = -X_2 e^{-\tau} N_1 \left( \ln \left( \frac{X_1}{X_2} \right) + \frac{(r - 1/2 \sigma_1^2) \tau}{\sigma \sqrt{\tau}} \right) \sigma \sqrt{\tau} \left( 1 - \rho X_1 X_2 \frac{\sigma_{x_1}}{\sigma_{x_2}} \right) \{\text{Equation 5}\}
\]

Where \( \sigma^2 \) is as defined in Equation 3 and \( N_1'(x) \) is the derivative of the standard normal distribution \( N_1(x) \) with respect to \( x \). The partial derivative produced in Equation 5 is negative when \( \sigma_2 > \rho_{12} \sigma_1 \).

Thus, if the variance of \( X_2 \) is large; the variance of \( X_1 \) is small and \( X_1 \) and \( X_2 \) are positively correlated, an increase in the volatility of \( X_1 \) can make the payoffs of \( X_1 \) closer to those of \( X_2 \) than they were before in most conceivable states and therefore decrease the payoff of exchanging one unit of asset \( X_1 \) for one of \( X_2 \).

In the particular context of this paper, Stulz’s discussion of the properties of volatility in this type of option may be important. The two assets of interest in this paper are \( X_G \), the present value of an average existing client of the relationship bank with more stable clientele, and \( X_A \), the present value client of average volatility and profitability of an average relationship bank. In the worst case, general volatility (\( \sigma^2 \)) adds value to a relationship when the independent variances of both own clients and industry clients are high, and the option is more valuable, satisfying the general condition \( \sigma^2 = \sigma_1^2 + \sigma_2^2 - 2 \rho_{12} \sigma_1 \sigma_2 \): In the best case, as demonstrated by Stulz’s argument above, while the volatility of an average industry client increases more than that of the ‘own’ client (while the correlation coefficient of the two clients is positive) or \( \sigma_A > \rho_{GA} \sigma_G \), the payoff of exercising the switching option described in this section will increase.

With the comparative statics that explain the value of volatility thus defined, a simulation of relationship value will be possible, provided reasonable estimates can be produced for the present value of expected cash flows from existing (\( x_G \)) and potential (\( x_A \)) clients, the maturity of the option and the cost of retention.

**Testing the Model**

The model produced for this paper was based on the self-reported switching intentions reported by respondents to a UK survey of 1,200 corporate clients conducted by Wood et al (1996).

For computational simplicity it was assumed that the option would have an eighteen month life. In practice, the switching option would have a life approaching perpetuity with appropriate levels of retention spending and no unforeseen structural changes.

As with Blattberg and Deighton’s model, retention costs (\( \delta \)) were included, and set at 0.5% of monthly cash flows. The choice of level was somewhat arbitrary although supported by anecdotal evidence provided by corporate bankers in informal interviews. In the discussion that follows the impact of varying this assumption is tested.
The present value of both the existing client relationship \((X_G)\) and the alternative, industry average client relationship \((X_A)\) were set equal at $100. By creating an ‘at the money option’ the value time value created by volatility and time remaining to maturity become the only sources of value in the option. Relaxation of this assumption is also tested below.

The contribution to switching option value contributed by the two sources of volatility is the main focus of this paper. 3.1% of the ‘good’ relationships bank’s existing clients self-reported intention to switch within three years \((\sigma_G^2)\) while the average switching intention for the five other relationship banks in the sample \((\sigma_A^2)\) was 7.3%. The correlation coefficient for the two subpopulations \((\rho_{AG})\) of 0.45 is a plug number, as the correlation of existing client switching intention and that of the industry is not available from the data, but general experience suggests that a positive correlation is likely. If correlation becomes too great (approaching 1.0) logically the value of the option will become worthless, since the possibility of value being created by switching disappears. As Stulz’s (1982) points out, a negative coefficient would lead to the greatest option value. The sensitivity of the model to this assumption is also discussed below.

The calculation of option volatility, based on the volatility of the two alternative clients and their correlation coefficient above, and Margrabe’s volatility equation first introduced in Equation 3 would be:

\[
\sigma^2 = \sigma_A^2 + \sigma_G^2 - 2\rho_{AG}\sigma_A\sigma_G
\]

\[
= (0.0780)^2 + (0.0508)^2 - 2 \times 0.45 \times 0.0780 \times 0.0508
\]

\[
= 0.0730 + 0.0310 - 2 \times 0.45 \times 0.0780 \times 0.0508
\]

\[
= 0.0051
\]

Applying monthly volatility of 0.51% to the Margrabe switching option formula produces the following option value:

\[
w(X_1, X_2, t) = e^{-0.05(18)}[100N(d_1) - 100N(d_2)]
\]

\[
\ln\left(\frac{100}{100}\right) + \frac{1}{2} (0.0051 + 0.005)(18)
\]

\[
d_1 = -\frac{\ln(100/100) + 0.0051 + 0.005(18)}{0.0714(18)} = 0.0909 = 0.3000
\]

\[
d_2 = 0.3000 - 0.0714(18) = -0.0029
\]

\[
N(d_1) = N(0.3000) = 0.6179
\]

\[
N(d_2) = N(-0.0029) = 0.4988
\]

\[
w(X_1, X_2, t) = e^{-0.05(18)}[100(0.6197) - 100(0.4988)] = 56.47 - 49.88 = 6.59
\]

From the calculations above, given the assumptions discussed above, the option has a value of $6.59. Because the option is a right to trade an asset with a present value of $100 for another worth $100, all of the value in the option comes from the volatility and time value.
Consistent with both Black and Scholes (1973) who identified the contribution of volatility and Stulz (1982) who provided a proof for the differential contribution of two sources of volatility, it would be expected that option value would increase as the total volatility of the two underlying assets increases: Both increases in the existing client’s relationship volatility and increases in the volatility of the average industry client relationships would increase the value contributed by total volatility, although the value of the option will increase more with an increase in ‘average’ industry volatility.

Recall that the estimated volatility of the industry average client was 7.3%, the average for the ‘good’ bank’s existing clients was 3.1%: As Figure 5 shows, as volatility in both assets underlying the option increases from these baseline values, volatility for each asset independently contributes incremental value to an at-the-money option, although the contribution from an increase in the volatility of the average industry client increases the option value more than that of the bank’s existing client.

**Figure 5  The contribution of existing and average industry client volatility to option value.**

![Graph showing the contribution of existing and average industry client volatility to option value.](image)

The value of the option rises as the volatility of the existing bank relationship approaches zero, consistent with the likelihood of exercise of the option decreasing. In order to isolate the contribution of volatility and test the sensitivity of findings to changes in the three other variables in the model (retention spending, ‘moneyness’ of the option and correlation of volatility terms) that required assumptions, the following sensitivity tests were applied.

**Comparative Statics of the Value of the Switching Option**

A full discussion of the sensitivities of model parameters was not possible here due to space limitations, but some key value drivers (such as retention costs, exercise price of alternatives, and correlation of options) deserve mention."
In estimating option value using the Margrabe model it was necessary to assume the cost of retaining an existing relationship. In order to isolate error introduced into the model by this assumption the level of retention spending was allowed to vary, with all other factors held constant. An increase in retention spending decreases the option value in an approximately linear fashion. When retention spending to support the existing client reaches 1.05% of the present value of relationship cash flows per month the option loses all of its value. Changes in retention spending have the effect of shifting both volatility curves in Figure 5 upwards for a decrease in retention costs, downwards for an increase in retention costs. A change of level of retention spending affects the value of the switching option, and it has no effect on the contribution to option value by the volatility of relationships.

The baseline assumptions tested above set the present value of both existing and alternative clients at $100, allowing all value contribution to be produced by the time-value contributed by volatility and the possibility of exercise value increasing in the eighteen months remaining until the option expires.

A test of the assumption that both options were ‘at the money’ was undertaken: As the present value of the alternative relationship increases, the intrinsic value of the option increases, and option value increases at an increasing rate as both intrinsic value and time-value contribute. As the alternative relationship value decreases the likelihood of in-the-money exercise of the option decreases, and the time value falls and intrinsic value is absent.

As discussed above, it was reasonable to assume that changes in the present value (arising from relationship switching intention) of existing client and alternative client relationships would be positively correlated. A correlation coefficient of 45% was used in the model, although there was no evidence that this value was correct. This assumption was tested, and as Stulz (1982) forecast, the contribution to option value by the existing bank relationship falls in relation to that of the alternative relationship as the correlation coefficient becomes a larger positive value. For negative or low correlation coefficients the change in option value for changes in existing relationship value tracks closely that of the alternative relationship albeit at higher option values. For uncorrelated or negatively correlated changes in asset volatility the option is more likely to produce an opportunity for an in-the-money exercise of the option before expiry. If asset price changes are highly correlated the opportunity to get ahead by switching becomes much less likely.

Conclusions

Estimation of the value of the ability to switch between an existing client relationship and an average industry one within the parameters discussed above was possible and produced results which were consistent with predictions of the model.

Increases in total relationship volatility produce value, but increases in average industry volatility create option value at a greater rate than volatility in the ‘own’ client relationship. Some volatility within the bank’s own client base is necessary in order for the option to become valuable, but the chances of alternative clients becoming even more attractive offers the greatest opportunity for option exercise.
In order for the model to work, certain assumptions were necessary, and of these assumptions, only the correlation coefficient for the variance of the two assets was significant. Random or negative covariance produced larger option values for the option, consistent with expectations. Positive, highly correlated volatility among the two assets reduced option value, and in extreme cases, an inverse relationship in the contribution to option value was noted: An increase in option value created by the existing relationship would offset the contribution by the industry average client. It is therefore reasonable to conclude that while the correlation of industry and own clients is significantly less than 1, the model as presented effectively demonstrates the contribution to relationship value of relative volatility.

References


